A CLASS IN GEOMETRY.

GEORGE ILES.

A Class in Geometry:

LESSONS IN OBSERVATION AND EXPERIMENT.

BY

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A CLASS IN GEOMETRY.

The Study of Geometry can begin with Observation.—The general disfavor with which my schoolfellows used to open Euclid is a vivid remembrance of my boyhood. In vain did the teacher say that geometry underlies not only architecture and engineering, but navigation and astronomy. As we were never given any illustration of this alleged underlying to make the fact stick in our minds, but were kept strictly to theorem and problem, Euclid remained for most of us the driest and dreariest lesson of the week. Not so with me; for geometry was my favorite study, and the easy triumph of leading the class in it was mine. As years of business life succeeded my school days I could not help observing a good many examples of the principles set forth in the lines and figures I had conned as a boy: examples which, had they been presented to my schoolfellows, would certainly have somewhat diminished Euclid's unpopularity. In fulness of time it fell to my lot to be concerned in the instruction of three boys, one fourteen years of age, the second twelve years of age, the third a few months younger. In thinking how I might make attractive to them what had once been my best-enjoyed lessons, I took up my ink-stained Euclid, Playfair's edition. A glance at its pages dispossessed me of all notion of going systematically through the propositions,—which took on at that moment a partic-

ularly rigid look, as if their connection with the world of fact and life was of the remotest. Why, I thought, not take a hint from the modern way of studying physics and chemistry? If a boy gets a better idea of an electro-magnet from a real electro-magnet than from a picture, or more clearly understands the chief characteristic of oxygen when he sees wood and iron burned in it than when he only hears or reads about its combustive energy, why not give him geometry embodied in a fact before stating it in abstract principle? Deciding to try what could be done in putting book and blackboard last instead of first, I made a beginning.

A House-lot and Two Fields tell us Much.—Taking the boys for a walk, I drew their attention to the shape of the lot on which their house stood. Its depth was nearly thrice its width, and it was surrounded by a low fence. As we went down the road, in a western suburb of Montreal, we noticed the shapes of other fenced lots and fields. Counting our paces and noting their number, we walked around two of those fields. This established the fact that both were square, and that while the area of one was an acre and a half, that of the other was ten acres. When we returned home I asked the boys to make drawings of the two fields, showing to a scale how they differed in size. [Fig. 1.] This task accomplished, they drew a diagram of the house-lot, and of a square equal in area to the house-lot. With a foot-rule passed around the latter diagram it was soon clear to them that if the four sides of the lot were equal, some fencing would be saved. [Fig. 2.] The next question was whether any other form of lot having straight sides could be enclosed with as little fence as a square. Rectangles and triangles were drawn in considerable variety and their areas computed, only to confirm a suspicion the boys had entertained from the first,—that of lots of practicable form square ones need least fencing. In

comparing their notes of the number of paces taken in walking around the two square fields, a fact of some importance came out. While the larger field contained

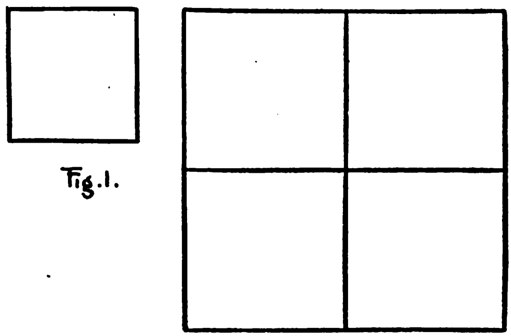


Fig. 1.

nearly seven times as much land as the smaller, it needed only about two and a half times as much fence to surround it. Taking a drawing of the larger enclosure, I divided

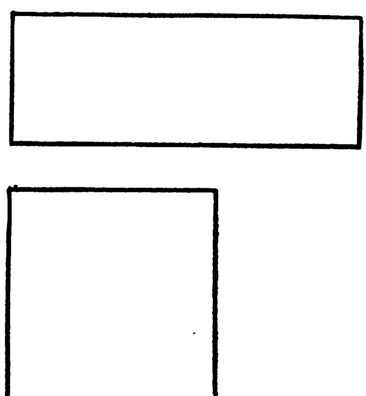


Fig. 2.

it into four equal parts by two lines drawn at right angles to each other. It needed only a moment for the boys to perceive that these lines of division, representing as they

did so much new boundary, explained why the small field had proportionately to area so much longer a fence than the large field. [Fig. 1.] A chess-board served as another illustration. With each of its sixty-four squares representing a farm duly enclosed, it was easy to see how a farmer who bought the whole number, were he to combine them in one stretch of land, could discard seven eighths of the fencing. During a journey by steamer from Montreal to Quebec, I directed the boys' attention to the disadvantageous way in which many of the farms had been divided into strips long and narrow. "Just like a row of chess squares run together," said one of the lads.

From Fact to Law.—When a good many examples had impressed our first lesson on their minds pretty thoroughly, I had the boys draw on large sheets of paper two squares, respectively 1 inch and 1 foot in length. Beneath these figures they wrote: "1 inch square has 4 lineal inches for boundary, 1 foot square has 48 lineal inches for boundary; 1 inch square has 1 square inch for area, 1 foot square has 144 square inches for area. Plane figures of the same form have boundaries varying *directly* as their like linear dimensions (length and breadth); they have areas varying as the *square* of their like linear dimensions." It proved, however, that while the boys knew this to be true of squares, they could not at first comprehend that it is equally true of other plane figures. They drew equilateral and other triangles, and in measuring their sides and areas ascertained that they conformed to the rule; but I was taken aback a little when the eldest boy said, "It isn't so with circles, is it?" His doubt was duly removed, but the remark showed how easy it is to make words outrun ideas,—how hard it is for a young mind to recognize new cases of a general law which in other examples is quite familiar.

Cinders Terrestrial and Celestial.—One chilly evening the room in which my pupils and I sat was warmed by a

grate-fire. Shaking out some small live coals, I bade the boys observe which of them turned black soonest. They were quick to see that the smallest did, but they were unable to tell why, until I broke a large glowing coal into a score of fragments, which almost at once became black. [Fig. 3.] Then one of them cried, "Why, smashing that

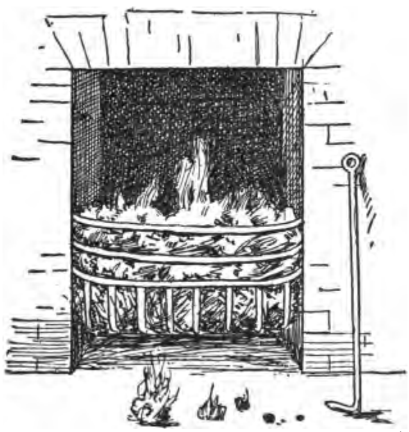


Fig. 3.

coal gave it more surface!" This young scholar was studying the elements of astronomy at school, so I had him give us some account of how the planets differ from one another in size, how the moon compares with the earth in volume, and how vastly larger than any of its worlds is the sun. Explaining to him the theory of the solar system's fiery origin, I shall not soon forget his keen delight —in which the others presently shared—when it burst upon him that because the moon is much smaller than the earth it must be much cooler; that, indeed, it is like a small cinder compared with a large one. It was easy to advance from this to understanding why Jupiter, with

eleven times the diameter of the earth, still glows faintly in the sky by its own light, and then to note that the sun pours out its wealth of heat and light because the immensity of its bulk means a comparatively small surface to radiate from. Here, incidentally, we came upon new ground for satisfaction with the small world we inhabit: only the surface of planets can sustain life, so that in proportion to its volume the earth bears eleven times the plants and animals it would were it swollen to the girth of Jupiter.

Cubical Blocks make the Law Plain and Clear.—To make the law concerned in these examples definite and clear, I took eight blocks, each an inch cube, and had the boys tell me how much surface each had—6 square inches. Building the eight blocks into one cube, they then counted the square inches of its surface—24: four times as many as those of each separate cube. With twenty-seven blocks built into a cube, that structure was found to have a surface of 54 square inches—nine times that of each component block. [Fig. 4.] As the blocks under-

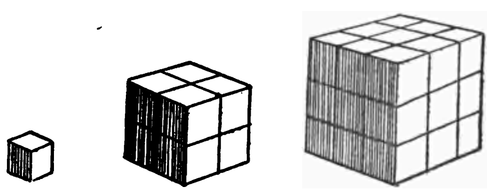


Fig 4.

went the building process, a portion of their surfaces came into contact, and thus hidden could not count in the outer surfaces of the large cubes. Observation and comparison brought the boys to the rule which told exactly what proportion of surface remained exposed. They wrote, "Solids of the same form vary in surface as the *square*, and in contents as the *cube*, of their like dimensions."

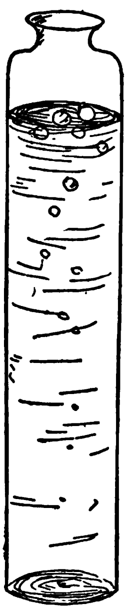
They were glad to note that the first half of their new rule was nothing but their old one of the farms and fields over again.

Gravity at the Surfaces of the Sun and the Earth.—Returning next day to the subject of astronomy, I took a terrestrial globe, and rotating it in a variety of planes made it clear that the centre of the sphere was the only point at rest in the mass, and that therefore the whole volume of a planet or a star may, in astronomical study, be considered as concentrated at the centre. The eldest boy was then asked to refer to his text-book and inform us what the radius of the sun is compared with that of the earth. He answered, about 109 times as much. He also found, on farther reference, that the specific gravity of the sun is about one fourth that of the earth. Taking the first figure, without fractions, he computed the volume of the sun in units of that of the earth as 1,295,029. He knew that gravity diminishes in intensity as the square of the distance through which it is exerted. I now inquired how much a mass weighing one pound on the surface of the earth would weigh if taken to the surface of the sun. Dividing 1,295,029 by 11,881 (the square of 109), and by 4 (the specific gravity of the earth as compared with that of the sun), he rendered 27¼ as the answer. With a little patience not only the eldest boy, but the other two boys, saw the analogy between this case and that of the large cube built out of the twenty-seven little ones. For a reason which would become clear in a future lesson, I asked them to ponder the principle that the very fact of bigness implies that the surface of an attracting mass is far from its centre, and that despite the immense rate at which the volume of a sphere increases as its diameter lengthens, a sphere's power of attraction at its surface increases only in the direct ratio of the diameter's increase of length,—as we had seen in ascertaining the comparatively small augmen-

tation of gravity which would be observable in a pound taken from the earth to an orb much more than a million times as big.

A Vial Illustrates the Path of an Ocean Steamer.—As the law at which we had now arrived is one of the most important in geometry, I took pains to illustrate it in a variety of ways. Taking a long, narrow vial of clear glass, nearly filled with glycerine, and corked, I passed it round, requesting each of the boys to shake it smartly, hold it upright, and observe which of the bubbles came to the surface first. [Fig. 5.] All three declared that the biggest did, but it was a little while before they could be made to discover why. They had to be reminded of the cinders and the building-blocks before they saw that the comparatively large surface of a small bubble retarded its motion through the liquid. The next day we visited the wharves of Montreal, and pacing alongside several vessels, jotted down their length. In response to questions, the boys showed their mastery of the principle which decides that the larger a ship the less in proportion to tonnage is the surface at which the vessel's motion is resisted. Going aboard an Allan liner, of five thousand tons burden, we descended to the engine-room. We next visited a steamer of somewhat less than one thousand tons and inspected her engines,—engines having proportionately to power much larger outer surfaces whereat to lose heat than those we had seen a few minutes before in the Allan steamship. When their experiments with the vial were recalled, the boys were pleasantly surprised to find that the largest bubble and the ocean racer come first to their respective ports by

Fig. 5.



virtue of their identical quality of bigness, by reason of the economies which dwell with size.

The Biggest Engines are Best.—But the ocean racer had an advantage in the bigness of its engines which was not to be read in a simple glance at their exteriors. The boys knew that in striking an anvil both anvil and hammer become heated, for at a blacksmith's shop near by they had had evidence of the fact; and in descending a slope of Mount Royal an empty cart had one day been observed checked in speed by a brake; a touch on this brake when the cart was stopped had informed them again that mechanical work can be converted into heat. Before we left the steamer I told the boys that if we had a thermometer connected with the cylinder of the engine we should see that the steam in expanding, in doing work, would fall in temperature: it stood to reason that if the power of the engine were applied, not in moving the ship but in lifting trip-hammers, the heat which these hammers would create in falling on anvils would be lost by the working steam. Only the eldest boy could follow me in this statement; it was some weeks before the others could do so; but all three were gradually brought to understand the great gain which comes of making a steam-cylinder as large as possible. I said that its contents in expanding become cooled, the steam thus cooled robs the cylinder itself of some of *its* heat, so that when the next working charge of steam comes in, that charge is chilled by the walls of the cylinder and loses part of its working power,—since working power depends on high temperature. Hence the importance of using the strongest material, as steel instead of iron, so as to make engines and cylinders of the utmost size, reducing, comparatively, the surfaces at which injurious cooling takes place. Before we left the steamer I added a word on the necessity for compactness in engines and machinery erected where space is scant, as aboard

ship, and told the boys that they would find the best engines for work on land modelled on the compact form of engines built for service at sea. (At the World's Columbian Exposition an Allis steam-engine of 4000 horse-power was shown. A firm in England is now, 1894, building a gas-engine of 650 horse-power, in dimensions hitherto unattempted. In the substitution of steel for iron an immense economy in weight is enjoyed by the builder not only of engines, but of machinery, buildings, and bridges. A similar, though less important, advance is promised in the current experiments with nickel and other alloys of steel and of aluminium. These alloys may yet give the world higher towers and longer bridge-spans, thinner and therefore more efficient boilers, and may, while providing larger engines and machines, render them capable of higher speeds than those at present safe.)

Balloons Small and Great. — One afternoon the boys and I while walking together espied a street-vendor with a supply of gaudy toy balloons. A balloon bought for the youngest of our party, I dare say that the little fellow was pretty confident that there was no Euclid in that plaything. It proved otherwise. That evening he calculated how much the lifting power of the balloon would gain on its surface were its dimensions multiplied one thousand or ten thousand fold,—step by step reaching the conclusion that if air-ships of that kind are ever to be manageable in the face of adverse winds, they must be made vastly larger than any balloons as yet put together.

A Store is Visited.—Not far from home stood a large store, displaying a miscellaneous stock of groceries, fruits, dry-goods, shoes, and so on. As we cast our eyes about its shelves, counters, and floor, we saw many kinds of packages—cans of fish, marmalade, and oil; glass jars of preserves and olives; boxes of rice and starch; large paper sacks of flour. Outside the door stood half a dozen empty

barrels and packing-cases. It certainly seemed as if the items of paper, glass, tin, and lumber for packages must enter largely into the cost of retailing. One after another the boys discovered that the store was giving them their old lesson in a new form: they saw that the larger a jar or box the less material, in proportion to capacity, it needed for its manufacture. The contents, too, of the jars, sacks, and boxes repeated the familiar story. The coffee had been ground from the bean that it might be the more effectively boiled; and certain brands of white sugar had been pulverized from the lump that they might be the more quickly dissolved at the breakfast-table. On their way home the boys were led to discern that form as well as size is an element in economy. Just as farms square in shape need least fence, so a cubical package needs least material to make it, and tins of cylindrical form require least metal when of equal height and breadth. (A matter this of some importance when we learn that during 1893 some 800,000,000 cans were made in the United States.)

The Builder needs Judgment in his Use of Geometry.—The boys well knew that of plane figures the circle, the section at right angles of a cylinder, is the one having least boundary in proportion to surface; they therefore understood why cylindrical pipes are used for water and gas. As we walked along we noticed here and there cylindrical conductors connecting the roofs of houses with the pavements. I bade the boys observe whether these conductors were in repair. In three or four instances they saw breaks and bulges in the metal due to the expansion of its contents in freezing. Here was a case where the cylindrical form was to be avoided, though usually chosen because the lowest in first cost. We noted that wherever a conductor had a square or corrugated outline, so as to permit the expansion of water in freezing, it had suffered no damage. [Fig. 6.] Clearly, then, the

question of judicious form was one depending upon circumstances to be carefully considered in each case. Of this we had another illustration when one morning we came upon a large ice-house near the river. It was very unlike a cube in its outlines, although a cube is the form bounded by planes which has least surface in comparison with capacity. The ice-house had a frontage twice the extent of its depth; its height was scarcely half its depth; its roof was not flat, but sloping. By a series of questions as to the strength of walls and the cost of building them so thick that they can be high; as to the expense of lifting and lowering ice in a lofty building; the necessity for making a roof aslant if it is to shed rain and melted snow, —we came to some of the reasons why the builder had de-

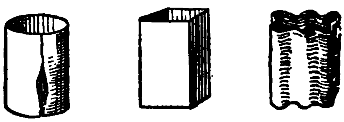


Fig. 6.

parted from the cubical form in this structure. But why was the ground plan oblong instead of square? Because the lot was of that configuration, and because that particular lot had advantages of locality,—it lay between the ice-field and the city, and was so near the city that the cost of cartage was minimized. About a mile from the ice-house stood a large sewing-machine factory. It also did not conform to a cubical outline: another series of questions brought out the desirability of its extended frontage for light, and of building a structure like this one, occupying costly land, of more stories than if it were built on cheap land. These examples were two of several which I gave the boys to demonstrate that it is a mistake to over-simplify a problem, and that judgment must direct

calculation and pass upon its results if a wise decision is to be reached. As they grew older they found that the common case of disagreement between practice and theory is due to theory not taking account of all the facts; that whenever a theory does take account of all the facts, any practice at variance with that theory is wrong, and should be abandoned.

Victoria Bridge.—Our next lesson, in somewhat the same line as the last, was one for lack of which not a few inventors and designers have wasted time and money. Taking the trio to Victoria Bridge, we inquired of the custodian the length of its central span. His reply was, 352 feet. When I asked the boys how matters would be changed if the span were twice as large in all three dimensions, they soon perceived that, while increased in strength by increase of breadth and thickness, it would be heavier by added length as well. On our return we compared two small beams differing in each of their three dimensions as 1 and 2, serving to make manifest why it often happens that a design for a bridge, roof, or machine, admirable in a small model, fails in the large dimensions of practical construction, and may even fall to pieces by its own weight. [Fig. 7.] For weight increases as the cube, and strength only as the cross-section, or square, of like dimensions.

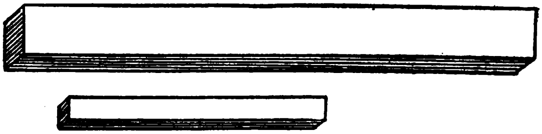
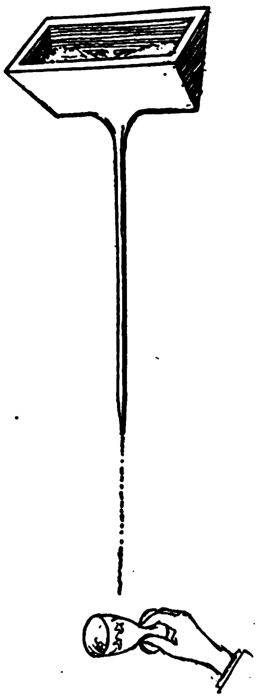


Fig. 7.

Why the Sand-blast is Efficient.—One day we went to a factory where designs were being marked upon glass by a sand-blast. The apparatus was of the simplest: from a hopper near the ceiling sand fell in a narrow stream

upon the panes or goblets exposed to its action. Had sandstone in lumps, say as large as playing marbles, been dropped upon the glass there would only have been harmful fracture. As it was, each particle of sand weighed too little in proportion to its striking surface to do any more than the desired work of detaching a tiny chip from the face of the glass it fell upon. [Fig. 8.]

Fig. 8.



Common Dust and Flour Dust.—Not long afterward a roofer was called in to make needful repairs at the house where two of the boys lived. We went with him to the roof, and found the gutter choked with mud. How had it got there? A glance at the roof—an iron one—showed it covered with dust which the next shower would sweep into the gutter. Dust particles are extremely small and fine—much smaller and finer than sand; and did not this explain how the wind had been able to take hold of them and carry them far up into the air? Here I said a word about the terrific explosions in the flour-mills of Minneapolis, due to nothing else than to flour reduced to a dust so fine as readily to form with air an explosive mixture. Disastrous explosions in the coal-mines of Austria have on investigation been proved chargeable to coal-dust as fine as powder, which has found its way through the air-shafts. When the boys afterward took up geology, they had a key to one of its most interesting chapters. Dust carried by winds has filled lakes,

covered plains, and buried cities from sight. Running over decayed rock, streams of water have borne its clay particles much farther than its sand.

Earth as Mud is easily carried by Water.—In early spring the rills and streams descending the slopes of Mount Royal are visibly charged with mud, as the boys had repeatedly observed. I told them that the Mississippi is not a clear river like the St. Lawrence, and gave them some of the surprising figures which record the quantity of earth in a state of division as fine mud which is borne away by the Father of Waters. Near its mouth the Mississippi sometimes carries in suspension as much as two pounds of solid matter in every cubic foot,—one thirty-second of its stream,—an amount so considerable that it enters into the computations of the geologist when he studies the principal ways in which the water of the globe gains upon the land.

Engineers by availing themselves of this carrying action of water are able to deepen watercourses by reducing to fine mud the surface of their beds. At Tilbury Dock, in England, this process has been adopted as only one fortieth as costly as ordinary dredging. It was found that in rapid eddies water could lift and remove clay, stone, and gravel, of much higher specific gravity than itself, when these obstructions were in small enough pieces.

Reduction to Powder often Advantageous.—In a new and improved way of manufacturing Portland cement, Mr. Ransome's, the ingredients are reduced to a powder instead of being brought together in lumps. The superiority of this method and its saving in time have long been hinted at in all the processes of grinding, as in the production of flour,—so much to be preferred to whole or even broken wheat. And be it remembered that in reducing wheat to powder the miller only imitates the action whereby his own teeth break his solid food into small, digestible

parts. As with wheat so with the fertilizers used in the wheat-field. A century ago bones merely broken were mixed with the earth by the farmer; to-day a fourfold efficiency arises from the application of bones ground to dust. And though very far from being as small as a particle of dust, it is partly because a grain of wheat or corn or rice is much smaller than ordinary fruits or vegetables—as apples or potatoes—that the cereals are man's best food. Their smallness enables them to dry quickly and thoroughly, so that they are easily preserved and transported.

How Rock becomes Soil.—In common with other similar sites in northern latitudes, Mount Royal in many scores of places in early spring shows how frost can detach the surface of a rock, or break into fragments a mass of stone into which water has entered. And an action very similar is exerted by the force of growth itself; we could observe in several places a sturdy sapling cleaving apart a not inconsiderable bit of rock, where a seed a few years before had taken root in a crevice. All this went to show the boys how hard rock gradually becomes soil by virtue of forces which break up large masses into smaller ones, and so expose new surfaces to dissolving air and water. Their attention was furthermore directed to the value of the plough, the harrow, and the sharp-toothed "cultivator," which lift and stir the soil of a farm or garden,—creating fresh surfaces whereat rootlets can feed, and rendering easy the growth of these thread-like structures.

Plants and Insects in Farther Illustration.— Aneighbor of ours had a large conservatory, in which, among other exotics, were various bristling cacti. When the boys learned that the cactus is a native of the desert, they saw at once its adaptation to dry air and a harsh soil; in comparison with ordinary plants its surface is meagre, and its stem very fleshy. This while according to Professor Asa

Gray an ordinary elm in full leaf has five acres of foliage. One afternoon our neighbor showed us a convolvulus he had just received from Colorado, presenting a curious phase of adaptation in a contrary direction. Its stem was that of a slender herb, yet in search for nourishment in a soil almost barren, its roots had extended themselves into fibrous cylinders so numerous as completely to fill a flour-barrel. Within the conservatory, as well as in the garden around it, there was abounding insect life. Since vital processes go on at surfaces, the smaller an aphis, midge, or beetle, the more reason, we could see, had the gardener to dread it, for the more food, proportionately to its weight, would it eat. The grasshopper, the largest of common field insects, easily springs to a height of three feet; its strength, proportionately so much greater than that of man, is due to its smallness, and therefore is exceeded by that of creatures smaller still.

Until within recent years the collectors and students of beetles, grubs, and moths were commonly regarded as people whose work was perhaps interesting, but of little practical account. To-day the economic side of entomology amply justifies to practical men the value, measured in money, of the science. At the Agricultural Colleges and Experiment Stations scattered throughout North America hundreds of naturalists are busy finding out how to combat the insect foes of the gardener and the farmer. Prof. Lintner, the State Entomologist of New York, has observed a larva which in twenty-four hours consumed 200 times its original weight; and a caterpillar which, in its progress to maturity, within thirty days increased in size 10,000 times. In view of the depredations of insects so voracious, is it any wonder that arsenic and other active poisons are now part of every intelligent farmer's stock-in-trade? Sometimes the farmer employs these poisons in warring upon very different pests, as, for example, the rot

which attacks the potato,—due to a fungus all the more formidable because extremely minute. The physician, like the entomologist, armed with a powerful microscope, tells us that minute organisms bring into the human frame consumption, diphtheria, and diseases equally serious. It would seem that the very smallness of these bacteria determines their rapidity in multiplication, their intense activity for evil. In another and less unpleasant branch of organic life the law we are considering comes within the purview of the naturalist. The late Prof. A. M. Marshall of Victoria University, Manchester, England, has pointed out that a change of form may be necessitated by an increase of size. Certain sea-slugs of the genus Limapontia are about one sixth of an inch in length; with a surface comparatively extended, they require no special organs for breathing, and their bodies are quite smooth. Slugs of allied species, but larger, have branching parts to breathe through, and these often take the form of fully developed gills,—a distinct advance in complexity of structure.

Adhesion as Depending upon Surface.—The boys well knew that in winter ashes strewn on icy streets prevent foot-passengers from falling; they had more than once remarked the efficacy of a little sand in enabling the wheels of a locomotive to start on an up-grade: they were to have another example of adhesion as a valuable property. In one of the rooms of the house were two pine tables, each 4 feet by 2, and 30 inches high. One of them had fluffy canton flannel tacked over it, and upon this flannel lay a well-worn table-cloth of linen, about four feet square. The other table stood bare. On gradually pulling the cloth off the first table it did not begin sliding to the floor until but one seventh remained on the canton-flannel table-top. On transferring the linen cloth to the bare table, it began to slide off when as much as one third of it rested upon the wooden surface. [Fig. 9.] A mag-

nifying-glass was then laid upon the linen, showing the boys how extensive the area of its fibres. [Fig. 10.] The

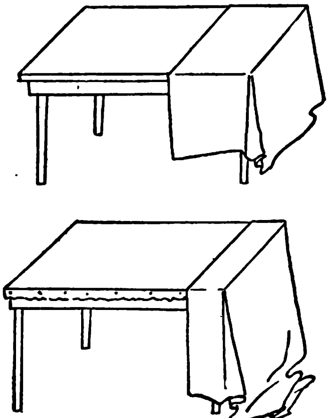


Fig. 9.

Fig. 10.

experiment with the tables was repeated by all three boys. They thus gained some idea of adhesion between surfaces of considerable length and breadth, and thereby apprehended an important aim of textile industry,—namely, giving adhesion its fullest play by spinning and weaving threads of the utmost possible surface. They were aided in understanding this by finding how much harder it was to pull out a thread from a skein of silk than to withdraw a strand from a similar skein of thick twine; while to pluck out a filament but two inches long from its place in the table-cloth proved an impossible task.

Motion Thermal, Electric, and Chemic.—At some distance from home, on the banks of the Lachine Canal, there stood a mill for the manufacture of doors, window-sashes, and other articles for the house-builder. We paid this mill a visit, and took occasion to observe how the motion of the piston-rod, delivered to the fly-wheel, was

thence diversified in the various departments of the factory. Here it appeared as the rotation of a planing-machine, there as the swift to-and-fro of a gang of saws, in another quarter as the continuous movement in a line, now straight, now curved, of a band-saw; elsewhere it took the form of screw-like advance in a boring tool, or of a volute in a cutter executing an ornamental design. The boys soon comprehended the ease with which suitable appliances produce any form of motion from any other. I wished them to grasp something more: so as we were leaving the factory yard I pointed to a long jet of condensed steam rising to the sky from an escape-pipe. Steam, I explained, when raised at high pressure, moves, as particles, with great violence; hence two or three pounds of steam striking against a piston hurl it forward with tremendous force: when freed, as at an escape-pipe, the particles rush forth, at first with the full pace of their interior bombardment, —the short, interrupted paths of their motion when shut in, becoming a long and straight line such as that of the steam-jet before us.

My pupils already knew that the motion of a hammer directed upon an anvil soon brings both hammer and anvil to a warmth unpleasant to the touch: reminding them of this, I told them that heat is the rapid motion of a body in particles too small for detection even with the microscope. "Could you imagine a bag of marbles flung from one side of a field to another, and then imagine each marble in swift rotation while the bag as a whole was at rest?" They could. Then they had grasped the distinction between the motion of a mass as a mass, and the motion of its parts as parts; between what, at a later stage of instruction, they would distinguish as molar and molecular motion. We had now come to a point of uncommon significance in our lessons, and I dwelt upon it with intent that it might be fully comprehended. I had the youngest

boy find the abacus of his early years, and slide it, on end, along the length of a large table, and then swiftly turn the beads on their wires: he thus illustrated the distinction upon which I was laying stress, as he also did in throwing a top into the air and afterward spinning it swiftly. "Now, boys," I said, "in a short time you will be studying the forces of chemistry, heat, and electricity; these forces are due to peculiar motions of the minute particles, atoms, or molecules which build up every substance in nature. All these motions, as you will duly learn, are related to each other somewhat as are the various motions you saw in the machinery of the lumber-mill. Men of science, however, are far from having mastered economical plans of converting one form of atomic motion into another: thus, in producing electricity from heat more than nine tenths of the heat is wasted. The main reason for so serious a lack of skill is that heat, electricity, and chemical attraction affect atoms of a minuteness so extreme that the orbits in which the atoms move have thus far eluded not only the observer, but the theorist. Sir William Thomson (now Lord Kelvin), has made various estimates of the dimensions of molecules in liquid and solid substances; these estimates, on grounds which you will in due time understand, present as an average the $\frac{1}{760000000}$ of an inch as the diameter of a molecule. Professor John Tyndall imagines 1 pound of hydrogen and 8 pounds of oxygen to fall from a height of 1100 miles to the surface of the earth. The momentum of such a fall would be enormous, would it not? Yet an equal momentum quietly resides in the gases as they exist side by side before they combine to form 9 pounds of water. The heat evolved on this combination is, as you shall one of these days see, vastly more intense than can be produced by any smiting of an anvil by a hammer, and yet it is due to the same cause, namely, collision. All we have to do, if we wish to

prove the correctness of Professor Tyndall's figures, is to observe the distance through which a hammer falls in descending to an anvil, and compare the heat of its collision with the heat set free when hydrogen and oxygen unite to form water. Every common gas-flame, indeed, bears witness to collisions at velocities much greater than that of a musket-ball—collisions which bring the molecules of gas to glowing heat."

"You are well aware that a grindstone or a fly-wheel will burst by centrifugal force if too swiftly turned; why is it that the atom, part of whose motion we have reason to believe is rotary, can turn immensely faster than a grindstone or a fly-wheel?" To answer this question I showed them a model consisting of two wooden wheels, each 6 inches in diameter and each covered with a rubber band to increase adhesion; both wheels, in contact, were fastened to a frame. Beside them was a wheel 12 inches in diameter. All three wheels were 1 inch in thickness. [Fig. 11.] The tendency whereby each wheel tended to

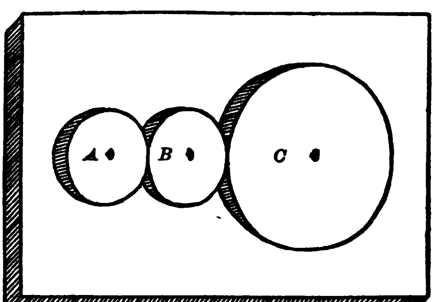


Fig. 11.

fly to pieces on swift rotation was ascertained by multiplying its mass by the square of its circumferential velocity, and dividing by the radius of the wheel. As *C* had 4 times the mass and only twice the radius of *A*,

the safe limit of speed for A was 1.41 (the square root of 2) times that for C. It thus became perfectly plain that if a wheel were as minute as an atom, it could safely withstand a marvellous swiftness of rotary motion. Our model taught us more. The peripheries of the two small wheels in the course of one rotation described two circles whose united length equalled that of the orbit described in one rotation of the rim of the large wheel,—and this while A and B had together but half the mass of C. Again imagining the small wheels reduced to atomic proportions, it was evident that in the transmission of motion by superficial adhesion, which may be the way in which electricity is transmitted, we were helped in trying to understand the surpassing velocity of electrical conduction. Were an inch cube to be filled with atoms such as those of Sir William Thomson's estimate, the eldest boy calculated that in comparison with the inch cube they would have, if cubical, a total superficies 760,000,000 times as great, or a surface of almost $1\frac{1}{4}$ square miles. The law of surfaces and volumes had taught us much with regard to masses; it brought us at last to the consequences involved in the infinitesimal dimensions of the units which form masses,—their astonishing capacity for motion, the amazing velocity whereat they can propagate motion.

The Cube and Pyramid; the Sphere, Cylinder, and Cone.—Our next lesson was intended to bring out the relations which subsist between several of the principal forms of solids. Two series of models in wood were accordingly obtained. The first consisted of a cube having a base 5 inches square, and a wedge and pyramid of similar base and height. [Fig. 12.] The second series comprised a sphere 5 inches in diameter, and a cylinder and a cone each 5 inches in breadth and height. [Fig. 13.] We took up the first series, when a moment's comparison of the sides of the wedge and cube told us that one contained

half as much wood as the other; but that the pyramid contained a third as much as the cube was not evident on

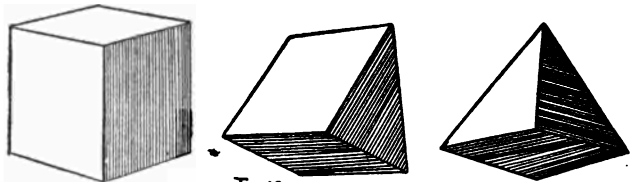
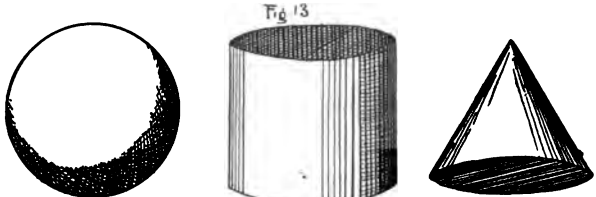


Fig. 12.

mere inspection. This relation was brought out in weighing the pyramid and cube; but a more satisfactory demon-

Fig. 13



stration was desirable, for what was to assure us that the two solids were of the same specific gravity? We now

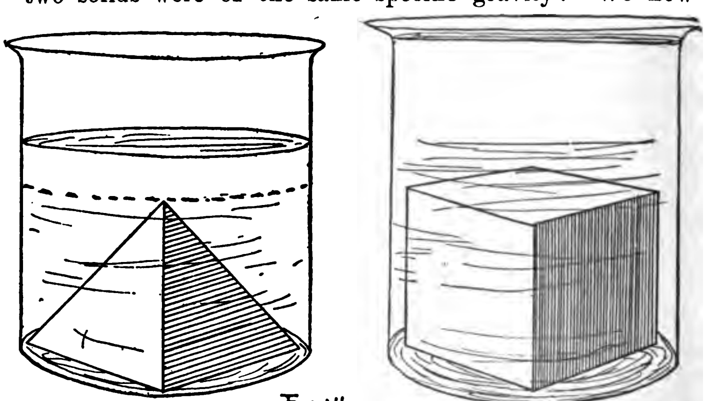


Fig. 14.

took a clear glass jar of cylindrical interior, measuring 7½ inches in width by 10 in height, and half filled it with

water. The models, duly varnished, and laden with lead so as to sink, were then successively immersed, and their displacement of the water noted with the aid of a foot-rule. This proved that the pyramid had one third the volume of the cube, that the same proportion subsisted be-

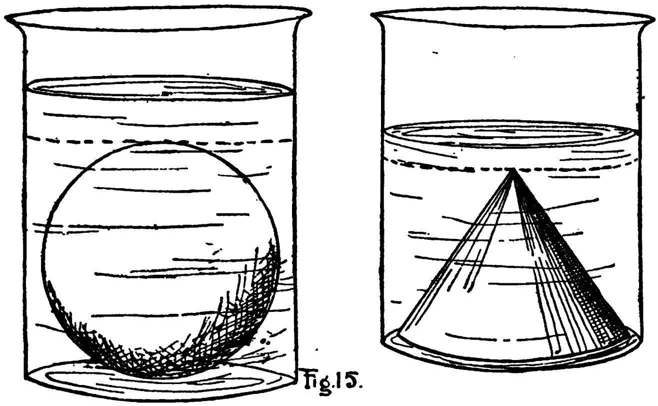


Fig. 15.

tween the cone and the cylinder, and that the sphere had twice the contents of the cone. [Figs. 14 and 15.]

A Square-root Extractor.—In a lesson given shortly after the formation of our little class (p. 6) it had been made clear that plane figures of the same form have areas varying as the square of their like dimensions. Dividing our wedge at a triangular side by three parallel lines, an equal distance apart [Fig. 16], I asked how the area of the smallest triangle, *HJC*, so laid off, and that of the next smallest, *FGC*, would compare with the area of the large triangle formed by the whole side of the wedge, *ABC*. "As the square of their sides," was the answer, the correctness of which was there and then confirmed. Dipping the wedge below the surface of the water in the jar, edge downward, we observed the water displaced as the square of the depth of immersion. When we reversed the process, the wedge be-

came a simple means of extracting square root. We had already divided the wedge into four parts by equidistant parallel lines; we now divided the vertical play of its displacement into sixteen equal parts marked along the side of the jar. Then, for example, if we sought the square root of 9, we immersed the wedge with its edge downward until it had displaced water to line 9, on the side of the jar; on the wedge the water stood at line 3, the square root of 9.

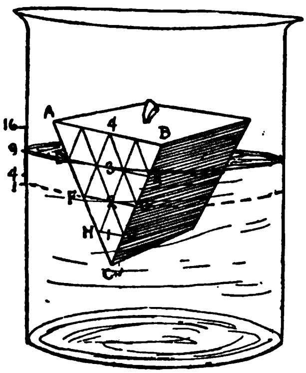


Fig. 16.

A Cube-root Extractor.—In one of our first lessons we had learned (p. 8) that solids of the same form vary in contents as the cube of their like dimensions. New proof of this rule also was at hand. We took a cone, and marking it off into three sections of equal breadth, we had within its surface three cones: first, the whole figure, *AGD*; second, a cone two thirds the height, breadth, and depth of the whole figure, *AEC*; third, a cone one third the height, breadth, and depth of the whole figure, *AFB*. All three were in strictness solids of the *same* form, so that their contents were in the ratios of 1, 8, and 27—the cubes, respectively, of 1, 2, and 3. The cone, as a whole, apex downward, we now immersed in the jar. It was observed to displace water as the cube of its depth of immersion,

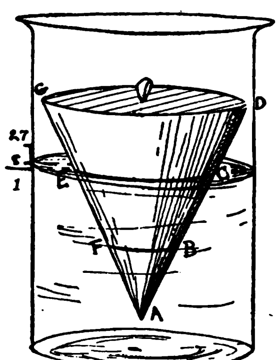


Fig. 17.

and therefore could be impressed into the service of extracting cube root. For this purpose its total play of displacement in a jar $5\frac{1}{2}$ inches interior diameter was divided into 27 equal parts; the cone was already marked off into three sections of equal breadth. [Fig. 17.] To find the cube root of 8, we lowered the cone, apex downward, until the water-level was brought to 8 on the side of the jar; at that moment the liquid encircled the cone at section 2, the cube root of 8. The pyramid immersed in the larger jar was also found to be available as a cube-root extractor. After our first experiments in extracting square and cube root by immersion, we found it advantageous to use as tall glass jars as we could get; the difference between high and low tide being thereby increased, we could make our scale on the side of the jar much longer, and therefore each of its divisions was more clear to the eye. We also found it helpful to have the cone descend from a rod turning in the screw-thread of a light framework. [Fig. 18.]

Fig. 18.

Perspective.—Measuring the cone and pyramid at each of their sectional divisions, the boys were required to ascertain the rule governing their increase of sectional area, arriving at the old familiar law of squares—a law true not only of all solids converging regularly to a point, but of all forces divergent or radiant from a centre, simply because it is a law of space through which such forces exert themselves. That the cone of our experiments represented the transmission of light from a

circle was observed by one of the boys during the projection of some photographs of scenery through a stereopticon in a public hall, where he drew my attention to the cones of rays shot forth from the exhibitor's lantern in the gallery—rays clearly defined in the dust of the air. More than once we had all noticed the beautiful effect in the sky known as "drawing water," due to solar beams reflected and scattered by a cloud into lines much resembling those which bound a pyramid.

I had a camera with which I occasionally permitted the eldest boy to take pictures. Arranging on an easel a sheet of paper, 4 feet square, I had him take its photograph. Then bringing the easel to a point half as far from the camera as at first, I had him photograph a sheet of paper 2 feet square. Its picture was of exactly the same size as that in the first experiment. I asked him to imagine lines drawn from both pieces of paper at their respective distances to their image in the camera; he soon saw that these lines would body forth a pyramid. We had here an explanation of how hands and feet, extended toward a camera by a sitter, are exaggerated in a photograph, especially when a common, simple lens is employed. From this we passed to some elementary consideration of perspective—an allied and timely theme, as the boys had

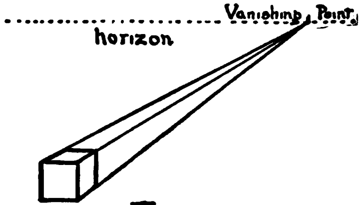


Fig. 19.

begun to practise drawing. Taking a picture of a box, I showed how its outlines, as far as they went, were but short

lengths, or parts, of the lines which bound a pyramid at its angles. [Fig. 19.]

Triangles and Circles.—We now addressed ourselves to the study of triangles, beginning by drawing them on paper, in acute-, right-, and obtuse-angled forms. We found that each of these triangles might be considered as half a parallelogram of equal base and height, so that with a foot-rule the computation of a triangular area was easy. We also found, by our sector, that the three angles of any triangle are equal to two right angles. Keeping distinctly in mind the three types of triangles—the acute-, the right- and the obtuse-angled,—we next proceeded to calculate how the square of the longest side, or the square of one of two equal sides, compares with the sum of the squares of the other two sides. At the end of a great many tests we concluded that only in the case of the right-angled triangle is the square of one side equal to the sum of the squares of the other two sides; and we found that the more nearly an acute- or obtuse-angled triangle approaches the form of a right-angled triangle the more closely does it exemplify this relation. [Fig. 20.] Formal proof of which relation, and of other relations that we had arrived at, awaited the

Fig. 20.

boys, I said, in the Euclid they were to study in due season. Before taking leave, for the time being, of our triangles, I had the boys cut in stiff cardboard a number of flat frames having 3, 4, 5, and more sides. Fastening these frames at the corners with pins, we found the triangular frame the only one not liable to fold down and collapse. [Fig. 21.] Here was a fact to be remembered, because by and by it would help to make clear why builders of roofs and bridges choose for their trusses triangular shapes, or the trapezoidal forms which share their stability.

Two members of my little class were able mentally to square numbers containing three numerals—by breaking up a number into two manageable parts, squaring the first, adding to this twice the product of the first and second, and then adding the square of the second. Taking a sheet

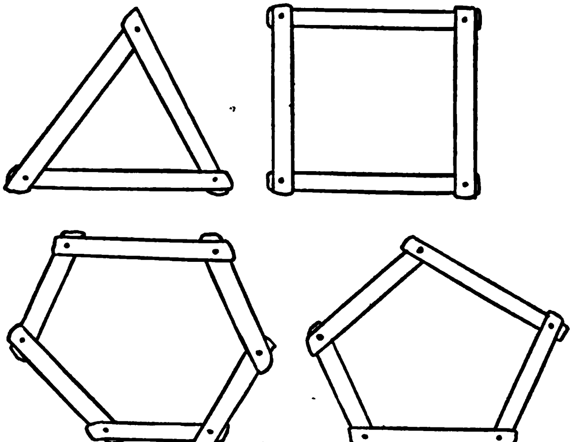


Fig. 21.

of paper we drew a diagram in illustration of the process. [Fig. 22.] After repeating the figure in outlines having a base divided in various proportions, I wrote beneath the illustrations,

$$A + B = C,$$
$$(A + B)^2 = A^2 + 2AB + B^2,$$

and explaining how the formula should be read, the boys began to see that algebra bears much the same relation to geometry and arithmetic that numerals do to numbers written out in words—that of an invaluable shorthand; a shorthand, too, which expresses relations in their widest generality.

While I was glad to employ observed examples and models in the instruction of my pupils, I wished them to grasp certain geometrical relations through exercise of the imagination. Reminding them that any triangle may be considered as half a parallelogram of equal base and height, we proceeded to the study of the circle. I told them the old way of ascertaining the area of a circle by conceiving it to be made up of an indefinitely great number of tri-

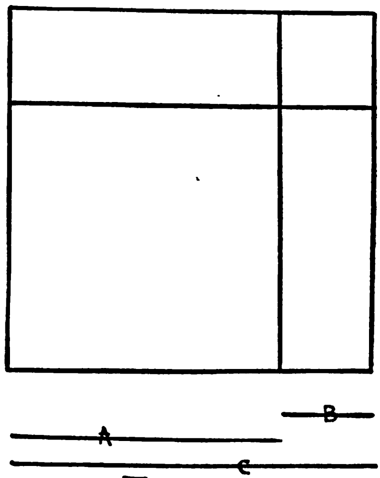


Fig. 22.

angles, whose bases become the circle's circumference, and whose altitude is the circle's radius. Rolling the cylindrical model round once on a sheet of paper, we marked off its circuit; this was made the base-line of a parallelogram having a height equal to half the cylinder's breadth; half that area was clearly equal to the surface of the circle forming the cylinder's base. [Fig. 23.] Another mode of indicating the relation between the circumference and the area of a circle was followed by the boys with fair prompt-

ness. I asked them to imagine a circular disk to be made up of the contact of a great number of concentric rings. Supposing the disk to be 1 foot in diameter, and each ring

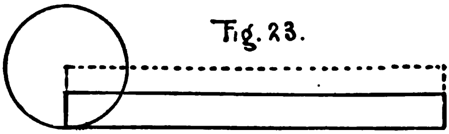


Fig. 23.

to be 1 millionth of a foot wide, I inquired, "How many rings would there be?" "Half as many—half a million," was the answer. To the question, "What would be the size of the circumference of an average ring?" "Half that of the whole circle," was the reply. The boys were thus brought to see that if a circle rolled round once is found to have 3.1416 lineal units for its circumference, its area must be .7854, or one half of one half as much, expressed in superficial units of the same order. [Fig. 24.]

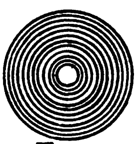


Fig. 24.

To come from circles abstract to circles concrete: I placed a map of Montreal and its environs on the table. With his house as a centre the eldest boy then drew a circle having for its radius 1 mile—the distance he could walk in 15 minutes. On his bicycle he could ride thrice as fast. "Now," I asked, "please draw a circle with a radius equal to the distance you can ride in 15 minutes." He did so, and, with the radius of his trip lengthened thrice, he found that with his wheel he had a circle of country at his command ninefold as extended as if he went afoot. To-day, a similar object-lesson on the value of rapid transit in extending the area of crowded cities might add to the bicycle the new suburban train-service which employs electricity for a pace much faster than that of the horse,—a pace limited only by considerations of safety.

Conical Surfaces.—The boys well knew that in dividing a pie by lines running from its centre each piece is just as large as the included arc of the circle makes it. With this fact in their minds I asked them to lay a cone on its side and roll it round once on a sheet of paper. The cone in its excursion, duly marked out, gave a sector of a circle, which, like the piece of pie it resembled, had an area proportioned to the length of its arc. On a former occasion we had found that the area of a circle is its circumference multiplied by half its radius: therefore the area of the sector before us was the length of its arc multiplied by half its radius. Recalling that this sector was equal to the curved surface of the cone which in rolling had described it, the boys saw the reason for the rule employed in ascertaining the curved surface of a cone —the multiplication of the circumference of its base (the arc of our sector) by half the slant height (half the radius of our sector). [Fig. 25.]

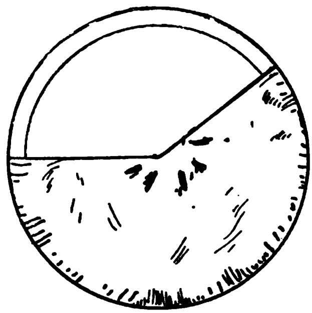
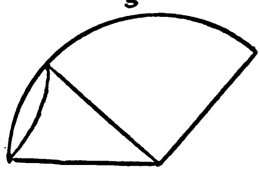


Fig. 25.



Great-circle Sailing.—At this stage in the course of our lessons there was much public interest in the Greely expedition to the North Pole: I thought that I would lead up to a consideration of that Arctic voyage by an illustration of great-circle sailing. Turning a terrestrial globe upon its axis, we observed that the Gillolo Islands and Cape San Francisco occupied points on the equator. A ship's shortest course plainly lay along the equatorial line which joined them. I asked which was the shortest

route from Portland, Oregon, to the northern extremity of the Japanese island of Yesso. The boys concurred in a wrong answer: "Along the parallel of northern latitude 45° 30′,—the parallel which joins them." Taking a brass-wire semicircle, D C, equal in diameter to the equator, and applying it so as to touch both places, the lads saw at once that the shortest route would take a ship somewhat toward the north for the first half of the voyage, and somewhat toward the south for the second half; that if two ports are to be joined by an arc, the largest circle of which that arc can form part marks out the shortest track, because it is the nearest possible approach to a straight line: furthermore, this largest or great circle is practically no other than a new equator cutting the earth in a plane inclined

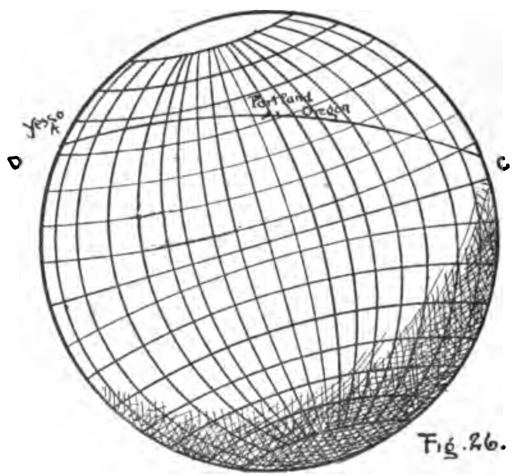


Fig. 26.

to the geographical equator. [Fig. 26.] We now proceeded to measure, in arcs of great circles, the distance from Port Nelson, in Hudson's Bay, to Liverpool: it proved to be a little shorter than the route from either Montreal or New York to Liverpool, simply because the sphere of

the earth contracts more and more as its poles are approached. Now we could understand how the possibility of finding an open polar sea dazzles the imagination of explorers, and lures them one after another to scenes of privation and death. Were navigation proved to be free across the Arctic circle, a new and much shortened route would at once be laid down between Asia and Europe, and Asia and the eastern coast of America. I was careful to point out that the navigator, like his friend the builder ashore, has to consider something besides the immediate indications of geometry. An ocean route which, at first view, seems most circuitous, is really the best when by choosing it a captain avoids adverse winds and currents, shoals and reefs, or the chance of meeting ice-floes and icebergs.

(The New York Central and Hudson River Railroad deflects considerably from a straight line drawn between its terminal points, New York and Buffalo; it is because of that deflection that the road enjoys its economy in working expenses. It is better for a railroad to make a moderate detour than to climb a hill for the sake of following a direct line; in conforming to the valleys of the Hudson and the Mohawk rivers the railroad named has easier gradients than those of any other line running westward from the Atlantic seaboard. Incidentally, too, the road, in passing through a succession of towns and cities originally founded on the Hudson and Mohawk rivers, has a business vastly larger than if it had been built in a straight line. This advantage in apparent indirectness is shared by nearly every other important railroad in America. In Russia, where military considerations outweigh those of trade and commerce, several of the principal railroads are as nearly straight as their engineering difficulties would permit.)

The Reading of Plans.—Not many years ago there dwelt in Albany, New York, a stove-founder, who had to

waste many thousands of dollars because he could not imagine from a drawing how the stove it depicted would look. In every case of a new design it was necessary to go to the large expense of building him an actual pattern,

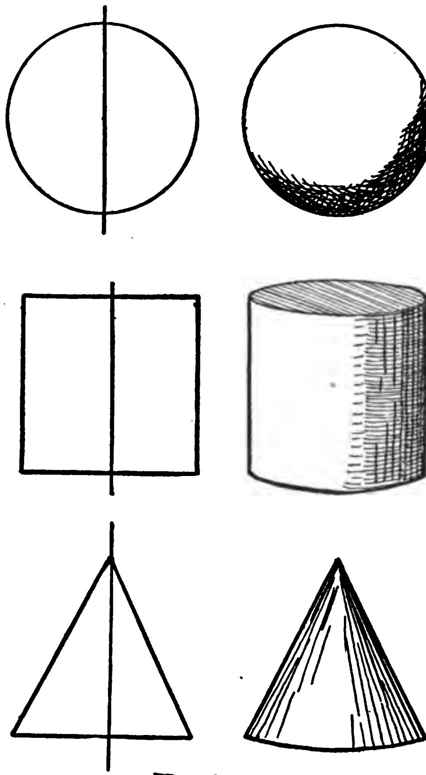


Fig. 27.

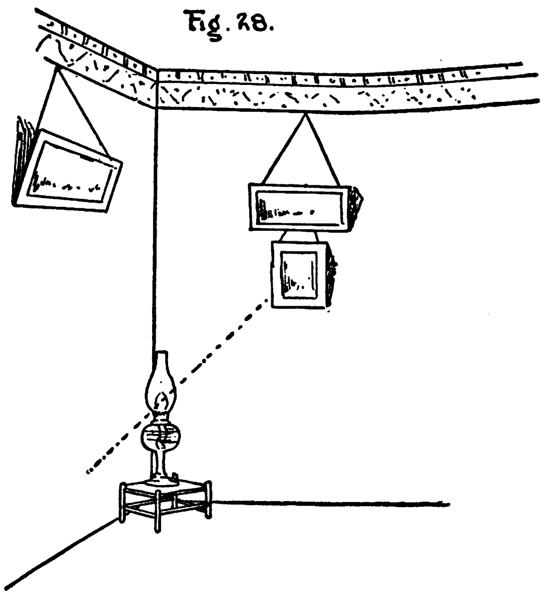
and these patterns had not seldom to be rejected as unsuitable. That my pupils might, if possible, be spared his incapacity, they were given a variety of exercises with solids and their sectional planes. Our 5-inch models of the

sphere, cylinder, and cone were each sawn in halves. By applying the cleft surface of each to paper, and having the resulting figure outlined, I made intelligible the generation of solids by the revolution of planes. The square, triangular, and circular outlines were then cut out of the paper; each was divided by an axis, and made to revolve around it. [Fig. 27.] Procuring a variety of lathe-work from a carpenter's shop,—hand-rails, table-legs, and the like,—the boys drew them in section, and these drawings were then compared with the woodwork sawn down its centre. In the main the accuracy of the drawings, after the first attempts, was commendable. A school provided with the tools for manual training could, of course, have much developed this elementary education of the eye. Without such a school the boys with a little practice were able to make plans of the houses they lived in, and to read similar plans published in a popular journal of architecture. Through lack of this easily cultivated power of being able to body forth how a house will look from its drawings, there is much endurance of discomfort, or a wasteful alteration of rooms, halls, and staircases, by people who have neglected to make their eyes stereoscopic, even in a rudimentary degree.

Proof of a Newtonian Theorem.—One evening all the lights in our sitting-room were extinguished except that of a small lamp. This lamp I placed in various positions,—in the centre of the room, near a wall, in a corner near the floor, and so on. At each place of momentary rest I asked the lads to imagine the flame bisected by any plane so extended as to divide the room into two parts. They were brought to see that no matter how small one of these parts might be, even if only a square foot or so in a corner near the floor, it received half the flame's light, because the bisecting plane sent half the rays into that little bit of corner. For the sake of illustration we assumed the area

of the bit of corner to be $\frac{1}{100}$ the area of the remainder of the room. As both these areas received equal quantities of light, it was evident that the geometrical mean distance of the larger section from the flame was 10 times that of the smaller. When the boys had mastered this thoroughly, they saw that nothing was changed if we imagined the lamp-flame reduced to the dimensions of a luminous par-

Fig. 28.



ticle, and that we were now free from having to disregard the shadows cast by the body of the lamp. But suppose a radical change of affairs,—the walls, floor, and ceiling to radiate light of uniform brilliancy, on a particle otherwise dark: would the particle receive on any two halves equal amounts of light? It took a good while for the boys to understand this case as the converse of the other,—to see that, according to this new supposition, if a plane divided

the room into parts having areas as 1 to 100, then such a plane would perforce bisect the particle at a geometrical mean distance of 10 from the larger section, and at a geometrical mean distance of 1 from the smaller section; that the particle when shone upon, instead of being radiant, would receive on any two of its halves precisely equal quantities of light. I now told the class that we had seized the substance of one of Newton's famous propositions. In his "Principia" he proves that if within a spherical shell of infinite thinness a particle is placed, which particle is attracted by the shell inversely as the square of the distance, then at whatever point it may be put the particle will remain at rest, the attractions on any two of its halves exactly balancing. Our luminous particle had not only showed this proposition to be true of spherical shells, but of shells—interior surfaces—of any form whatever, provided they have no re-entrant angles relatively to the particle. Indeed, any shell not spherical may be regarded as made up of points in a series of spherical shells expanding out from an enclosed particle as a centre,—the points being exposed to form the continuous surface presented to the particle. All this, I told the boys, would apply when they took up the study of light and heat, and came to the theory of exchanges. They were now ready to advance another step in the use of light as a means of making visible certain laws of geometry.

A Sphere and its Enclosing Cylinder.—I asked them to imagine a spherical shell lighted by a particle at its centre. They apprehended at once that the shell would be uniformly illuminated. "Now," I said, "suppose the particle replaced by an axis evenly shedding forth beams equal in amount, but in directions at right angles to itself, the shell will receive exactly as much light as before, will it not?" "Of course." "Now let us suppose the spherical shell to be transparent, and enclosed by a cylindrical one having

the same height and breadth, will not the cylinder also be uniformly illuminated throughout?" "Yes." "And with the same brightness as the sphere in our first supposition?" "Yes; the radii will have equal length in both cases." "Well, then, the curved surface of that cylinder must have precisely the same area as the sphere, for both receive an identical amount and brightness of light." Not

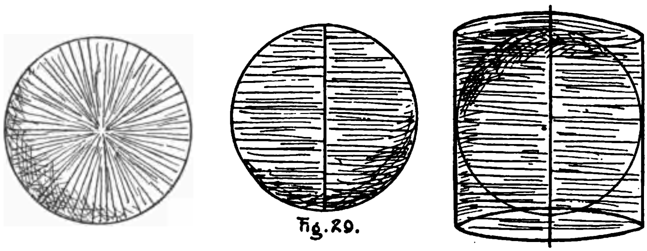


Fig. 29.

so promptly came the comment, "So it must." [Fig. 29.] I said to the boys that if they kept all this in mind it would by and by enable them to understand how the curved surface of the earth, globular as it is, can be represented on the flat surfaces of maps and charts.

Here let me say, parenthetically, that an important point has since occurred to me in connection with that lesson—the sphere in the second supposition is uniformly illuminated from its axis exactly as when the light streams forth in all directions from its centre. Thus there is new proof of a familiar method of finding the area of a spherical zone or segment—the enclosing a sphere within a vertical cylinder and producing the parallel horizontal planes which bound the segment until they cut off a section of the cylinder; the area of that cylindrical section is obviously equal to the area of the spherical segment. In other

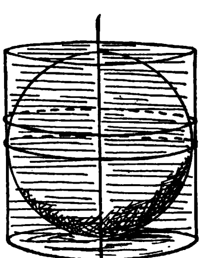


Fig. 30.

words, the zone's surface is equal to the zone's altitude

multiplied by the circumference of a great circle of its sphere. [Fig. 30.]

The Lessons Incite to Independent Observation.—About a year had now elapsed since the formation of our little class, and our progress was gratifying. The eldest boy had begun the study of Euclid at school, and was earning uncommon marks for his proficiency. In the lessons I have recorded, and in others which followed them, all the lads showed their interest by being constantly on the look-out for new illustrations. Let an instance or two of this suffice. One day they walked to an immense sugar-refinery some distance off, paced around it, estimated its height, and brought me their calculations as to its storage capacity in comparison with that of a small warehouse near by,—calculations showing how much outer wall and roof were saved in the vast proportions of the refinery. At home an extension of the house was heated in winter by a small stove; at a neighboring station of the street-railway there was a much larger stove of the same pattern. Counting efficiency to depend on surface, one of the boys asked me if it would not be better to have two small stoves instead of the large one. He was perfectly conversant with the reason why steam-fitters use small pipes for their heating-coils, and why their radiators abound in knobs and ridges. He saw, for himself, why in buying a tin baking-oven one of cubical form had been chosen—he knew that an oven of that shape would throw out wastefully and harmfully less heat than if it were made in any other form. One morning our neighbor's conservatory had more than a dozen of its panes of glass broken by hailstones. My eldest pupil came to me with the reason why the hail had been so destructive. Certain of its pellets had cohered to form masses of ice nearly half an inch in diameter. These masses, less resisted by the air, and heavier than separate pellets, had fallen with the force of bullets. One summer

day the boys and I came to a hay-field; part of the hay was being made, and part had been built into stacks. They pointed out that the farmers in making hay tedded it out to get all the light and air it could, and that when the hay was thoroughly dry it was kept from rain and frost in compactly built stacks of small exposed surface.

Laws are Strings wherewith to tie Facts together.—It may be no more than the effect of bias due to an individual preference for the study, but, in the light of its influence on these three young minds, I cannot help thinking that geometry affords a happy means of developing and directing the powers of observation and reasoning. Our lessons, simple as they were, had shown us that a single elementary principle such as that subsisting between surfaces and volumes may bind together a wide variety of fact in physics, chemistry, astronomy, navigation, and engineering. Young people are apt to imagine that a most formidable barrier rises between science and common things: we had discovered that no such barrier exists. When the boys came to study minerals, plants, and insects, they found that they could gainfully adopt the method with which they had taken up geometry—their collections had value, and were added to with ever-renewed interest, because they kept in mind a connecting thread of classification, instead of accumulating at random a variety of merely curious things. One of the boys gathered specimens of materials used for building in the adjoining city, and could show limestones and sandstones as they leave the quarry and are prepared by the mason with hammer and chisel. His collection included bricks burned and unburned, mouldings in terra cotta, ornamental glazed tiles, and specimens of the rarer cabinet woods.

Invention.—William George Spencer, the father of Herbert Spencer, in a little book entitled "Inventional

Geometry" has shown how geometry can be taught so as to educe the noble faculty of invention. At the High School in Yonkers, New York, I have seen original and most beautiful solutions of Mr. Spencer's problems worked out by the pupils. I doubt not that in other fields of study a lively interest can be aroused by keeping before learners an important principle of nature or art—and inciting to the search for illustrations of it. Such a principle may serve as a finder-thought in clearing away a difficulty not otherwise to be surmounted. Originality, ingenuity, can thus be drawn out, adding to the resourcefulness of a student's mind. At the High School in Yonkers and at other excellent schools the literary exercises make an appeal to inventiveness, and not less effectively than the lessons in geometry. A teacher tells a story to the extent of one half; the remainder has to be written from the imagination of the listeners. This method is fully illustrated in Professor E. R. Shaw's recently published "English Composition by Practice." In several newly established schools of industrial art there is a somewhat similar challenge to ingenuity. A teacher draws a few straight lines and simple curves upon the blackboard, and after a minute or two rubs them out. His pupils then repeat them on their slates from memory, and combine them to form original designs. All this is good. It makes for the development of individual aptitude. It gives a pupil a pleasing sense of having, in however small a way, added to knowledge or to art; while, incidentally, the knowledge or the art is the better grasped and cemented in the act of making this addition, inconsiderable though it be. Besides the direct gain which comes of bringing out the ingenuity of boys and girls in these ways and in all ways, there is an indirect gain not less important—in laying the foundation for an intelligent sympathy

with the labors of the inventor, the discoverer, the man of originality who finds a new and good way out of a difficulty industrial, social, or political. Only when all the people are informed as to the gifts these men stand ready to confer, only when they are heartily willing to receive those gifts, can civilization come to its best estate.